

Active Disjunctive Constraint Acquisition

Grégoire Menguy, CEA LIST, France

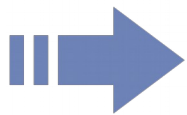
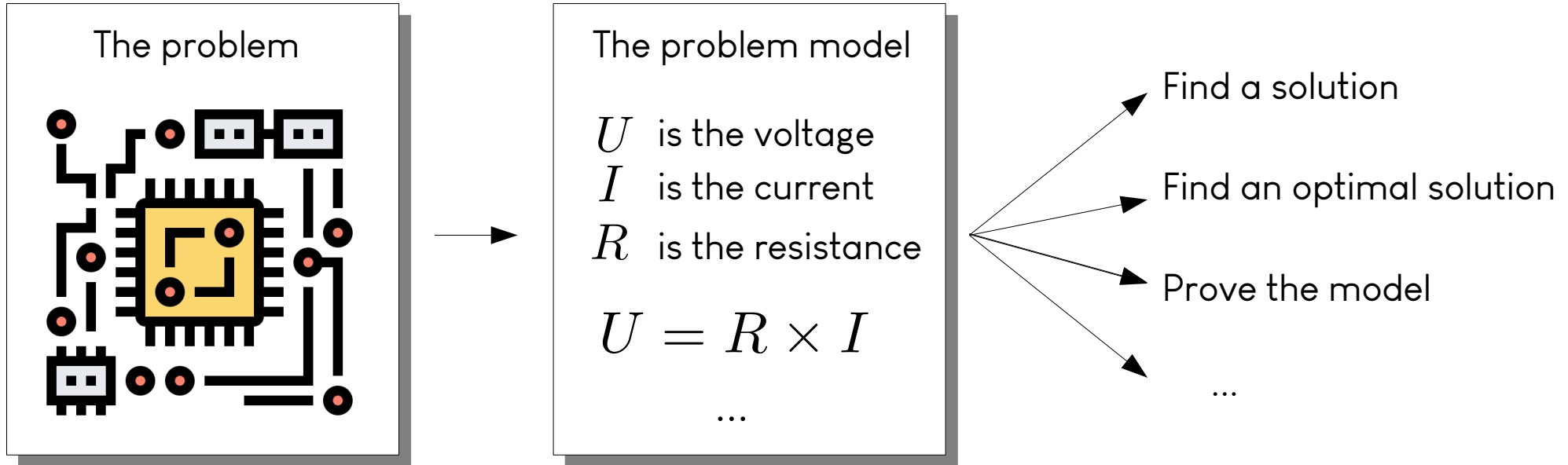
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Arnaud Gotlieb, Simula, Norway

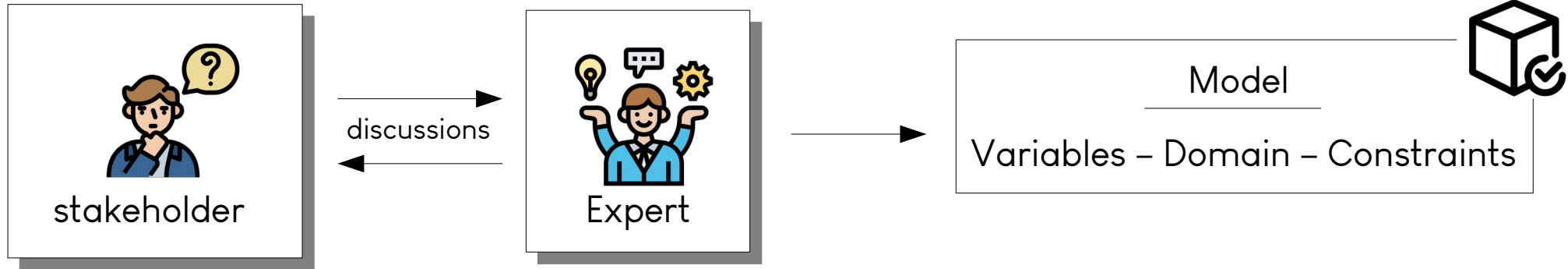


Solving problems with constraints



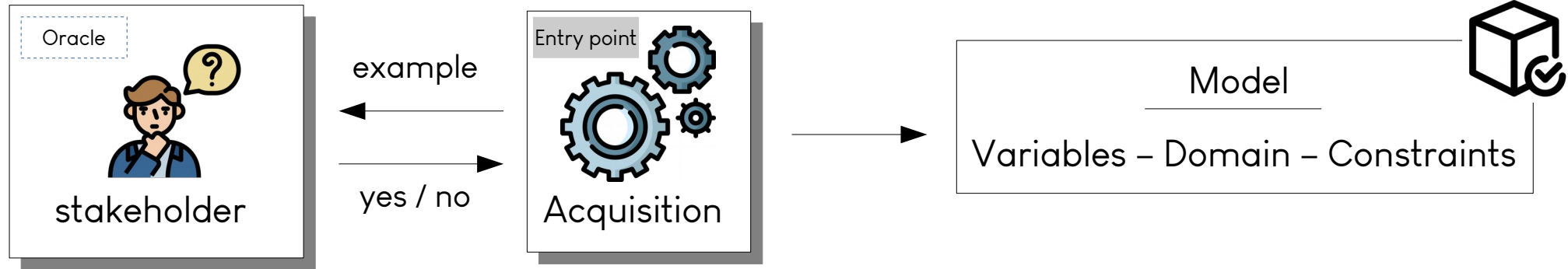
But where does the model come from ?

Modeling is hard



- ↳ Modeling needs expertise
- ↳ How to assist the stakeholder ?

Constraints Acquisition



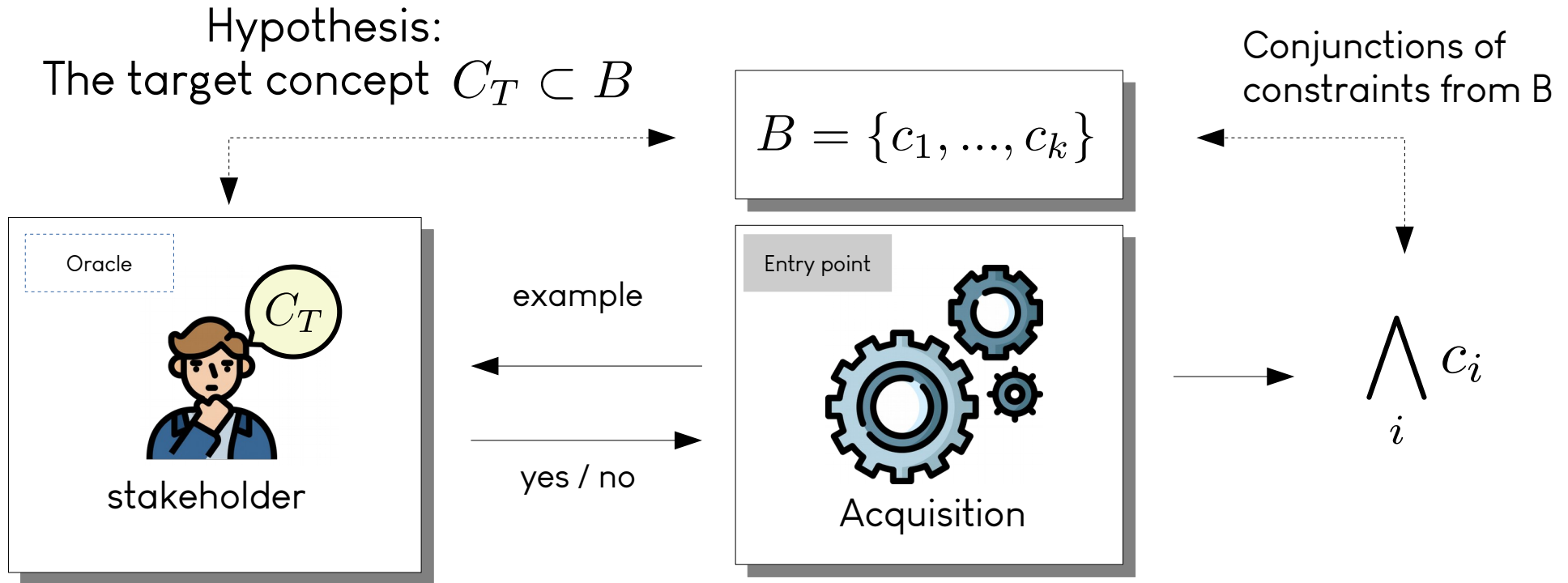
Different approaches:

- Inductive Logic Programming based [Lallouet et.al., ICTAI10]
- ModelSeeker [Beldiceanu and Simonis et.al., CP12]
- **Conacq based on Version space learning [Bessiere et al. 2017]**
 - ↳ Formal foundations
 - ↳ Passive and active learning

Conacq guarantees:

- ↳ Termination
- ↳ Queries are informative
- ↳ Correctness

Problem: CA is conjunctive only ...



But Most Problems Need Disjunctions

For example

- Program analysis applications like Precondition acquisition (Menguy et al. 2022)

```
int sum(int *arr, uint size) {  
    int res = 0;  
    for (uint i = 0; i < size; i++)  
        res = res + arr[i];  
    return res;  
}
```



To execute without crashing the
function input should be s.t.,
 $size = 0 \vee valid(arr)$

- Biology application with the ultra-metric constraint (Moore and Prosser 2008)

$$(X > Y = Z) \vee (Y > X = Z) \vee (Z > X = Y) \vee (X = Y = Z)$$

- and many more

So how to handle it?

We need
 $size = 0 \vee valid(arr)$

- Rely on expert knowledge

↳ Adds exactly the good disjunctions

But we do not have such expertise – otherwise, why using CA ?

```
int sum(int *arr, uint size) {  
    int res = 0;  
    for (uint i = 0; i < size; i++)  
        res = res + arr[i];  
    return res;  
}
```



- Add all disjunctions with size up to some threshold

↳ Which threshold ?



↳ Combinatorial explosion leads to the bias size explosion

So how to handle it?

– Rely on expert

↳ Adds expert

But we do not
why using C

– Add all disjoint

↳ Which to

↳ Combin

Our key ingredients

– Maximally Satisfiable Subset* (MSS)

↳ No threshold i.e., no need for an expert

↳ No bias size explosion

↳ MSS enumeration is well established
with well studied algorithms

* will be defined later

```
size) {  
    for (size; i++)  
        r[i];
```



Contributions

- First extension of CA to the disjunctive case
 - ↳ Extension of representability to $\vee$ -representability
- Show how MSS can be used for concept learning
- Propose DCA and show its guarantees
 - ↳ Same as usual CA: Termination, informativeness of the queries, correctness
- Evaluation DCA
 - ↳ Outperforms CA based inference (faster and handle more complex concepts)
 - ↳ Applied on program analysis – handle complex functions like mbedtIs

New Expressiveness Hypothesis

NEW

C_T is a conjunctions of constraints from B , i.e.,

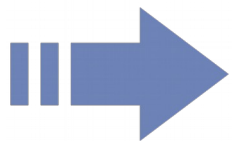
$$C_T \subset B$$



C_T is a conjunction of disjunctions of B 's constraints, i.e.,

$$C_T = \{d_i\}_{i \in I}$$

$$d_i = \bigvee_j a_j \quad \text{with } a_j \in B$$



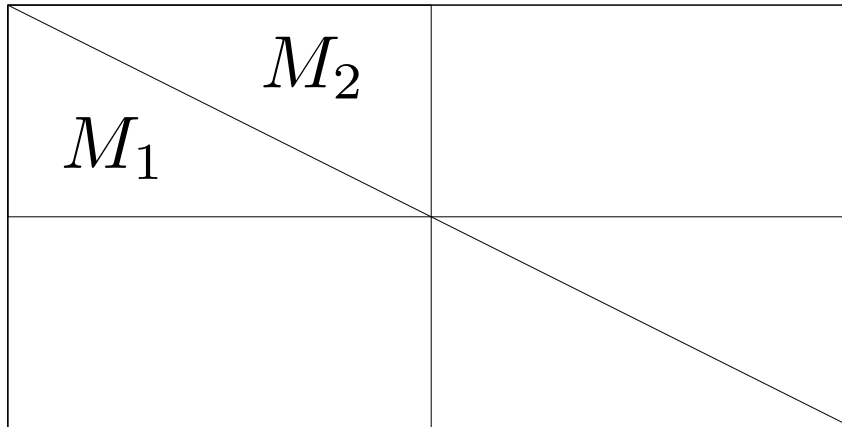
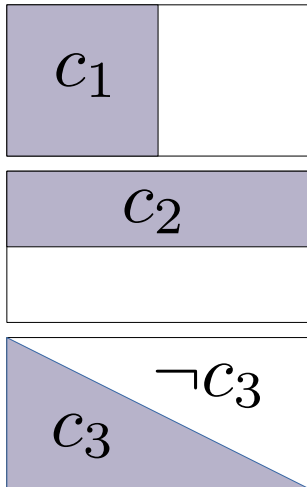
We say that C_T is $\bigvee$ -representable by B

Maximal Satisfiable Set (MSS)



Definition : Let B an unsat bias, then $M \subset B$ is an MSS of B iff M is sat but for all $c \in B \setminus M$, $M \cup \{c\}$ is unsat.

Example:



$$M_1 = \{c_1, c_2, c_3\}$$

$$M_2 = \{c_1, c_2, \neg c_3\}$$

Maximal Satisfiable Set (MSS)



Definition : Let B an unsat bias, then $M \subset B$ is an MSS of B iff M is sat but for all $c \in B \setminus M$, $M \cup \{c\}$ is unsat.

Example:

↪ We have a partition of the domain

c_1	$\neg c_1$
c_2	
$\neg c_2$	
c_3	$\neg c_3$

M_1	M_2	M_3
M_6	M_5	M_4

$$M_1 = \{c_1, c_2, c_3\}$$

$$M_2 = \{c_1, c_2, \neg c_3\}$$

...

$$M_6 = \{\neg c_1, \neg c_2, c_3\}$$

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Example:

c_1	$\neg c_1$
-------	------------

c_2
$\neg c_2$

$\neg c_3$
c_3

Property

Let B be a complete bias.

Then the set of MSS induces a partition of the domain

M_6	M_5	M_4
-------	-------	-------

$$M_6 = \{\neg c_1, \neg c_2, c_3\}$$

Classification of MSS

Property : If C_T is $\vee$ -representable by the complete bias B
Then all the solutions of an MSS share the same classification

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Property : If C_T is $\vee$ -representable by the complete bias B
 Then all the solutions of an MSS share the same classification

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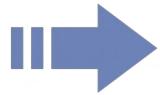
M_1	M_2	M_3
M_6	M_5	M_4

$$\begin{aligned}
 M_1 &= \{c_1, c_2, c_3\} \\
 M_2 &= \{c_1, c_2, \neg c_3\} \\
 &\dots \\
 M_6 &= \{\neg c_1, \neg c_2, c_3\}
 \end{aligned}$$

Disjunctive Constraint Acquisition

Key points :

- ↳ MSSes induce a partition
- ↳ Check classification of one element per MSS



Deduce the classification of all the domain

Algorithm 1: DCA

In : A nonempty complete bias B

Out : A conjunction of disjunctions of constraints from B

```

1 begin
2    $L \leftarrow \top$ 
3   foreach  $M \in \text{MSS}_B$  do
4     pick  $e \in \text{sol}(M)$ 
5     if  $\text{ask}(e) \neq \text{yes}$  then
6        $L \leftarrow L \wedge \neg M$ 
7   return  $L$ 
  
```

Theoretical analysis

Proposition : DCA generates informative queries only

Remark: Informativeness must be extended because of disjunctive behaviors. Some queries which were not informative in CA is informative now.

Theorem : If C_T is $\vee$ -representable by B then DCA infers a constraint network L s.t., $L \equiv C_T$



Terminates, result agrees with all tested queries

Theoretical analysis

Proposition : DCA generates informative queries only

Remark: Informative queries which were

riors. Some

Theorem : If constraint ne

DCA have the same good properties as CA



A infers a

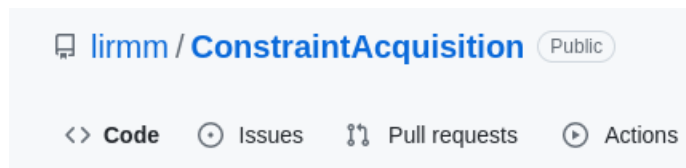


Terminates, result agrees with all tested queries

Implementation



↳ Extension of the Conacq.2 framework



Constraint acquisition [☆]

Christian Bessiere ^{a,*}, Frédéric Koriche ^b, Nadjib Lazaar ^a, Barry O'Sullivan ^c

↳ MSS enumeration algorithm: DAA vs MARCO

Enumerating Infeasibility: Finding Multiple MUSes Quickly

Mark H. Liffiton and Ammar Malik

Evaluation: Datasets

↳ Benchmark 1

- Random - e.g., $(X1 = X2 \vee X0 = X2) \wedge (X1 \leq X2 \vee X0 > X2) \wedge (X1 > X2 \vee X0 \geq X2)$
- Domain - e.g., $(X = 1 \wedge X_1 = 1 \wedge X_2 = 0 \wedge X_3 = 0) \vee (X = 2 \wedge X_1 = 0 \wedge X_2 = 1 \wedge X_3 = 0) \vee (X = 3 \wedge X_1 = 0 \wedge X_2 = 0 \wedge X_3 = 1)$
- Ultra-metric - e.g., $X > Y = Z \vee Y > X = Z \vee Z > X = Y \vee X = Y = Z$

↳ Benchmark 2

Automated Program Analysis: Revisiting Precondition Inference through Constraint Acquisition

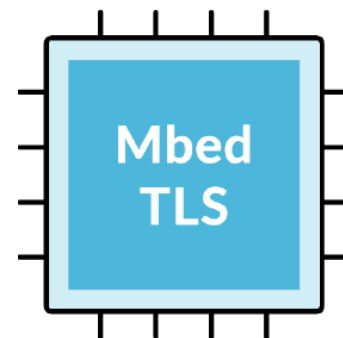
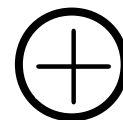
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²LIRMM, University of Montpellier, CNRS, Montpellier, France

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More complex
&
More disj.



Alternative approaches

Benchmark 1

- ↳ Conacq: includes in B all disjunctions of size up to the maximal needed
- ↳ Conacq_{Omniscient}: includes only disjunction size needed

Benchmark 2 (precondition inference)

- ↳ PreCA: as presented in the paper
- ↳ PreCA $|disj| \leq t$: includes disj. of size up to a threshold “t”
- ↳ PreCA_{Omniscient}: includes only disjunction size needed

Remark: Queries are answered using an oracle (the code under analysis itself for precondition inference)

Results Bench 1

↳ DCA is faster for Disj > 1

↳ DCA is not impacted by disjunction size
– But Conacq bias size explodes

↳ DCA handles more complex cases (e.g., UM₅)

Notations:

- $|B|$ is the number of constraints in the bias (for Conacq it includes disjunctions but not for DCA)
- $|E|$ is the number of queries submitted

	Disj	CONACQ			CONACQ _{omiscient}			DCA		
		$ B $	$ E $	Time	$ B $	$ E $	Time	$ B $	$ E $	Time
RAND _{2,1}	1	6	3	0.2s	6	3	0.3s	6	3	0.2s
RAND _{2,2}	2	18	3	0.4s	18	3	0.3s	6	3	0.2s
RAND _{2,3}	3	26	3	0.4s	14	3	0.2s	6	3	0.2s
RAND _{2,4}	4	26	3	0.4s	6	3	0.2s	6	3	0.2s
RAND _{3,1}	1	18	7	0.5s	18	6	0.3s	18	13	0.9s
RAND _{3,2}	2	162	13	5s	162	13	3.7s	18	13	0.9s
RAND _{3,3}	3	834	13	154s	690	13	43s	18	13	1s
RAND _{3,4}	4	2850	13	817s	2034	13	140s	18	13	0.9s
RAND _{4,1}	1	36	14	1s	36	15	1s	36	75	38s
RAND _{4,2}	2	648	54	286s	648	37	47s	36	75	39s
RAND _{4,3}	3	7176	-	TO	6564	-	TO	36	75	36s
RAND _{4,4}	4	56136	-	TO	48996	-	TO	36	75	37s
DOM ₃	3	834	24	297s	690	24	217s	18	24	0.6s
DOM ₄	4	9968	-	TO	7944	-	TO	24	64	1.2s
DOM ₅	5	122026	-	TO	96126	-	TO	30	160	3.3s
DOM ₆	6	-	-	TO	-	-	TO	36	384	9.7s
DOM ₇	7	-	-	ME	-	-	ME	42	896	44s
DOM ₈	8	-	-	ME	-	-	ME	48	2048	233s
DOM ₉	9	-	-	ME	-	-	ME	54	4608	1690s
DOM ₁₀	10	-	-	ME	-	-	ME	60	-	TO
UM ₃	4	472	13	50s	252	13	13s	12	13	0.5s
UM ₄	4	9968	-	TO	7944	-	TO	24	75	3s
UM ₅	4	87440	-	TO	77560	-	TO	40	541	200s
UM ₆	4	-	-	TO	-	-	TO	60	-	TO

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		CONACQ			CONACQ _{omiscient}			DCA		
		Disj	$ B $	$ E $ Time	$ B $	$ E $ Time	$ B $	$ E $ Time	$ B $	$ E $ Time
RAND _{2,1}	1	6	3	0.2s	6	3	0.3s	6	3	0.2s
RAND _{2,2}	2	18	3	0.4s	18	3	0.3s	6	3	0.2s
RAND _{2,3}	3	26	3	0.4s	14	3	0.2s	6	3	0.2s
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UM ₆	4	-	-	TO	-	-	TO	60	-	TO

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	<i>Disj</i>	CONACQ			CONACQ _{omiscient}			DCA		
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UM ₅	4	87440	-	TO	77560	-	TO	40	541	200s
UM ₆	4	-	-	TO	-	-	TO	60	-	TO

Results Bench 2 (precond. inference)

	Min bias				Avg bias				Max bias			
	1s	5s	5 mins	1h	1s	5s	5 mins	1h	1s	5s	5 mins	1h
PRECA	34/60	45/60	48/60	48/60	32/60	44/60	46/60	46/60	24/60	36/60	44/60	45/60
↳ No disj	21/60	21/60	21/60	21/60	21/60	21/60	21/60	21/60	20/60	21/60	21/60	21/60
↳ $ disj \leq 2$	38/60	43/60	44/60	44/60	35/60	42/60	44/60	44/60	21/60	38/60	44/60	44/60
↳ $ disj \leq 3$	30/60	44/60	48/60	48/60	26/60	43/60	46/60	46/60	18/60	31/60	42/60	44/60
↳ $ disj \leq 4$	30/60	43/60	48/60	48/60	26/60	42/60	45/60	46/60	18/60	29/60	35/60	40/60
↳ $ disj \leq 7$	30/60	43/60	48/60	48/60	27/60	42/60	45/60	45/60	18/60	28/60	35/60	35/60
↳ $ disj \leq 10$	30/60	43/60	48/60	48/60	27/60	42/60	45/60	45/60	17/60	27/60	35/60	35/60
↳ <i>Omniscient</i>	38/60	45/60	48/60	48/60	34/60	44/60	46/60	46/60	26/60	40/60	43/60	45/60
DCA	40/60	45/60	51/60	54/60	38/60	45/60	49/60	51/60	31/60	42/60	47/60	51/60

↳ Over each bias DCA infers in 5mins more preconditions than PreCA in 1h

↳ DCA is even more efficient than PreCA_{Omniscient}

Results Bench 2 (precond. inference)

	Min bias				Avg bias				Max bias			
	1s	5s	5 mins	1h	1s	5s	5 mins	1h	1s	5s	5 mins	1h
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↳ $ disj \leq 2$	38/60	43/60	44/60	44/60	35/60	42/60	44/60	44/60	21/60	38/60	44/60	44/60
↳ $ disj \leq 3$	30/60	44/60	48/60	48/60	26/60	43/60	46/60	46/60	18/60	31/60	42/60	44/60
↳ $ disj \leq 4$	30/60	43/60	48/60	48/60	26/60	42/60	45/60	46/60	18/60	29/60	35/60	40/60
↳ $ disj \leq 7$	30/60	43/60	48/60	48/60	27/60	42/60	45/60	45/60	18/60	28/60	35/60	35/60
↳ $ disj \leq 10$	30/60	43/60	48/60	48/60	27/60	42/60	45/60	45/60	17/60	27/60	35/60	35/60
↳ <i>Omniscient</i>	38/60	45/60	48/60	48/60	34/60	44/60	46/60	46/60	26/60	40/60	43/60	45/60
DCA	40/60	45/60	51/60	54/60	38/60	45/60	49/60	51/60	31/60	42/60	47/60	51/60

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↳ DCA is even more efficient than $\text{PreCA}_{\text{Omniscient}}$

An example from MbedTLS



```
int mbedtls_md_finish(mbedtls_md_context_t *ctx,  
    unsigned char *output);
```

Description: Delete structures of selected MD

Postcondition: $Q = \text{"ret"} = 0$

```
typedef enum {  
    MBEDTLS_MD_NONE=0,      /**< None. */  
    MBEDTLS_MD_MD5,         /**< MD5 */  
    MBEDTLS_MD_SHA1,        /**< SHA-1 */  
    MBEDTLS_MD_SHA224,      /**< SHA-224 */  
    MBEDTLS_MD_SHA256,      /**< SHA-256 */  
    MBEDTLS_MD_SHA384,      /**< SHA-384 */  
    MBEDTLS_MD_SHA512,      /**< SHA-512 */  
    MBEDTLS_MD_RIPEMD160,   /**< RIPEMD-160 */  
} mbedtls_md_type_t;
```

DCA result:

- #queries: 416
- convergence time: 401s
- Solution (simplified):

$$\begin{aligned} & \text{valid}(ctx) \wedge \text{valid}(md_info) \wedge \\ & \neg \text{alias}(ctx, md_info) \wedge \text{valid}(md_ctx) \wedge \\ & \left(\begin{array}{l} type = 1 \vee type = 2 \vee type = 3 \vee \\ type = 4 \vee type = 5 \vee type = 6 \vee type = 7 \end{array} \right) \\ & \wedge \text{valid}(output) \end{aligned}$$

Discussion

- Why DCA is faster than Conacq ?
 - ↳ DCA only relies on SAT and CP solving
 - ↳ Conacq also relies on Pseudo boolean solving
- Including a background knowledge
 - ↳ DCA can integrate a Background knowledge as a set of MUS
 - ↳ But experiments showed no impact
- DCA returns hard to understand results
 - ↳ Needs to simplify the results

Conclusion

- ↳ First extension of constraint acquisition to the disjunctive cases
 - New acquisition hypothesis : extension to $\vee$ -representability
- ↳ MSS enumeration for concept learning
 - MSSes induce a partition
 - A partition share the same classification
- ↳ DCA shows promising results on real-world instances and standard academic problems
 - e.g., application to precondition inference in program analysis

Leads to DCA

Same guarantees than best $\wedge$ -approches

↳ termination

↳ queries informativeness

↳ correctness



The background of the slide is an abstract composition of various shades of blue, ranging from light sky blue to deep navy blue. These colors are arranged in a complex, overlapping pattern of sharp, angular shapes, primarily triangles and polygons, creating a dynamic and modern geometric aesthetic.

End

Thank you for your
attention